

GEORGIA INSTITUTE OF TECHNOLOGY



Multi-point Optimization of Airfoils

Undergraduate Research Thesis

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Abstract

Many previous studies focused on developing optimum airfoils for steady flight conditions. However, with minimal extensions airfoils could be designed to perform better at the take-off conditions which would result in the efficient take-offs at shorter runways. An inverse design technique called the Modified Garabedian McFadden (MGM) technique was applied to NACA 0012 airfoil which resulted in an airfoil with drag bucket at the normal flight operation conditions. A newly developed optimization technique was applied to the three-element take-off configuration and a configuration that produced higher lift was obtained. Future work could be conducted on extending these design techniques for various airfoil configurations.

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Introduction

The cross section of the aircraft wing called an “airfoil” is often used to design and analyze the performance of the wing. Efficient airfoil shapes, measured in terms of the lift and drag forces produced, result in better aircraft performance such as longer range, take-off at shorter runways, reduction in fuel costs, etc. Aerodynamically, an optimal airfoil shape produces high lift and low drag within the design constraints often imposed by the structural requirements. The most general form of an airfoil (used on most commercial airplanes) consists of three individual units: slat, main element, and the flap. Each part has its importance in obtaining the required performance from the airfoil. Slat and flap are often deployed or retrieved based on the phase of the flight. Slat is used to delay stall such that an increment in the angle of attack doesn't cause adverse effect on the lift. The flap is used to increase the camber of the airfoil so that additional lift is obtained. Figure 1 summarizes the typical configuration of the wing at different phases of flight – level flight, take-off, and landing.

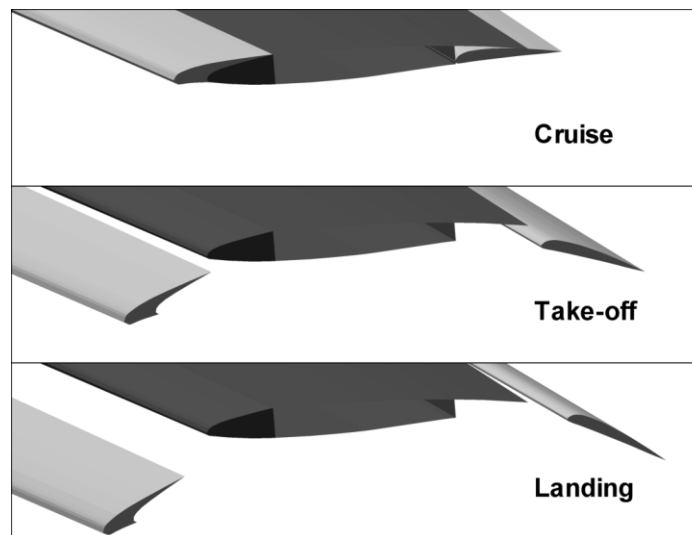


Figure 1: Wing configurations at different flight phases [14]

In the cruise phase i.e. when the slat and flap are retracted, the multi-element airfoil can be simplified (by ignoring the small gaps between the surfaces) to a single element airfoil. The

simplified single element airfoil's aerodynamic properties are often used to design an optimum wing cross section. Often, landing could be ignored because it is easily achieved through aileron deployment. Therefore, the two phases of flight that govern the airfoil design are the steady flight and take-off conditions. In most studies, the optimization process is applied to the cruise level condition while ignoring the take-off conditions. The results often result in inefficient take-off conditions which result in excess fuel procurement. Therefore, it is important to design the airfoil for both the steady-flight and take-off conditions.

Literature Review

Airfoil optimization has been a popular research topic for the past two decades. However, design of optimal airfoil shapes for multiple flight conditions has been studied by only few researchers. The purpose of the literature survey is to:

- Review previous studies on airfoil design and optimization. Investigate if those deficiencies can be covered with current knowledge and technology
- Find easily applicable design technique(s) optimization techniques that can be applied to both the single element and multi-element configurations collectively or separately.

American Institute of Aeronautics and Astronautics (AIAA) database provided several papers published on airfoil optimization processes. Among those articles, a study by Eric Besnard and Adeline Schmitz [3] provided a standard definition of the design procedure. According to the article, optimization is a three step process consisting of:

1. The representation of a configuration of by a set of design variables
2. The optimization method
3. The evaluation of aerodynamic performance for the new configuration.

The following pictorial representation of the process provided a better understanding of the procedure.

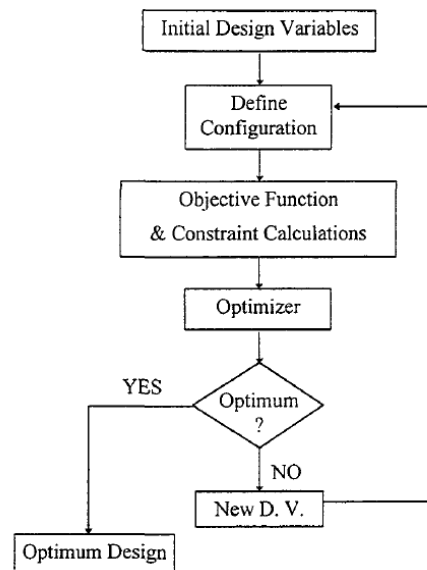


Figure 2 A pictorial description of the optimization process [3]

The article also presented an optimization method that was applied to high-lift devices. Application of their procedure requires higher mathematical knowledge and better computational resources. However, the article pointed at an important problem with application of Computational Fluid Dynamics (CFD) at that time. The absence of a reliable flow solver, which could accurately predict the viscous effects, turbulence, etc that are often seen in practicality, reduced the empirical application of their optimized design solutions. Similar studies by Eyi and Chand [5] achieved optimized designs. However, the practical application of their solutions was also limited by the unreliability of their CFD tools. From these studies it was evident that reliable flow solvers were big issues in previous airfoil designs. With the availability of cutting-edge CFD tools in the modern era, the errors between empirical and simulated results have become negligible. Therefore, the use of reliable flow solvers such as XFOIL for single-element airfoils and FLUENT for the multi-element configuration would provide a remedy to the deficiency in

not only the studies described above, but similar studies from the previous decades. The fidelity of the current generation flow solvers is shown in the results section.

Many of the airfoil design features conducted in the past used robust genetic algorithms to produce an optimum solution. The application of these algorithms requires advanced knowledge in formulation, highly accurate simulation tools and advanced computational resources [7]. In order to avoid such burdens, many studies often use inverse design techniques to meet the design requirements. Inverse design methods use the design requirements as target parameters, and base line configuration as the input. The design procedure creates the output design by modifying the baseline such that it meets the design criteria. This has become one of the popular design methods for airfoils due to its simplicity and efficient use of computational resources. Numerous inverse design procedures were formulated by researchers for different kinds of airfoils. Michael Selig has conducted numerous studies using such techniques [6, 12]. One of the popular techniques developed by him is the “Generalized Multipoint Inverse Airfoil Design” [12]. In this technique, the airfoil is divided into numerous segments along which the velocity requirements and other design parameters such as thickness are imposed. By using a modular design tool which couples an incompressible potential flow inverse design method with an integral boundary-layer analysis method, the desired airfoil was obtained. This tool could also be extended to compressible flows easily. The shortcomings of this method are that it requires very accurate information about the boundary layer conditions which may not be available to the designer, and extension of this method to multipoint optimization requires significant amount of resources.

An important research in the field of multipoint optimization of airfoils was conducted by Venkataraman at Rochester Institute of Technology [13]. Similar to the topic of interest, he

developed optimization techniques that could be applied to airfoils at steady-flight and take-off conditions. Although a detailed procedure of his design wasn't obtained, their results could be used to analyze the solutions from the current design procedure. His results showed that the resulting airfoil from a symmetric airfoil is thicker with an increase in the camber at the nose and lowering of the lower surface.

Research Objectives

The objectives of this research project were the following:

- I. Apply an inverse design technique to a single element configuration to minimize drag at the normal flight operating conditions
- II. Apply an optimization procedure to a multi-element configuration to maximize lift at the take-off conditions

Methodology

For the single element airfoil case i.e. the wing cross section at the level flight condition, an inverse design method called the Modified Garabedian McFadden (MGM) technique is applied [10, 11]. In order to optimize the performance of the three-element airfoil at the take-off conditions, a newly developed simple optimization technique is used. This section presents the theory behind these methods and the general procedure that can be applied to any airfoil.

Modified Garabedian McFadden (MGM) Technique

The MGM technique is an inverse design method that can be applied to airfoils and wings given a target pressure distribution. It was originally developed by Garabedian and McFadden for use in a very specific application wing design in a code called FLO22. It was

extended for use with any analysis (Panel, CFD) and any configuration (wing, airfoil, fuselage, etc.) by Malone and Sankar [10, 11].

The principle behind this method deals with the relation between the surface pressure distribution and the surface slope dZ/dx (where Z is the surface ordinate) and the curvature d^2Z/dx^2 . Changes in the pressure or speed between present values and target values will depend on the changes to Z , changes to slope, and the changes to the second derivative. Figure 3 shows the dependency of the target velocity on the ordinate, slope and curvature of the airfoil.

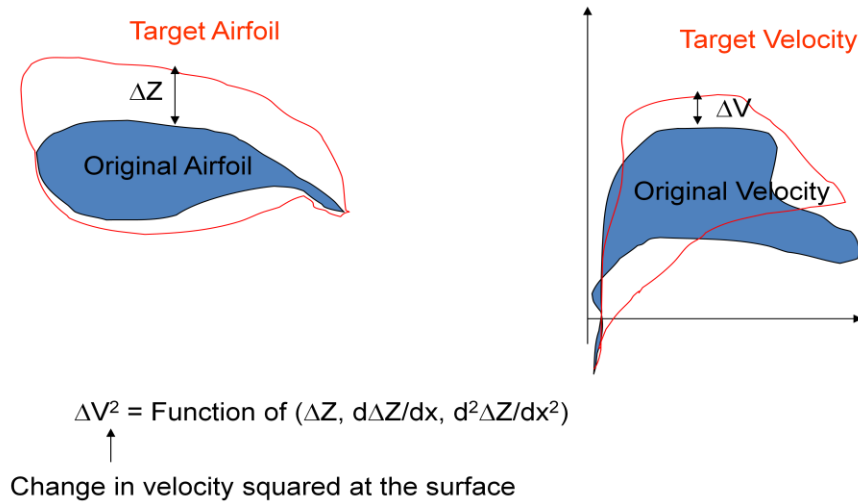


Figure 3: Explanation of the MGM technique

The relation between the Pressure Coefficient (C_p) and non-dimensionalized velocity just outside the boundary layer is obtained using Eq. (1):

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 \quad \text{Eq. (1)}$$

Substituting Eq. (1) into the function and simplifying further would result in the following equation:

$$A * \Delta Z + B * \frac{d(\Delta Z)}{dx} + C * \frac{d^2(\Delta Z)}{dx^2} = V_{target}^2 - V_{present}^2 \quad \text{Eq. (2)}$$

where A, B & C are arbitrary constants which constrain the airfoil from changing too much. Even though the C_p on the airfoil is a more complicated function of the surface slope, curvature and ordinates, to decide how the ordinates of the given airfoil should be changed, the above equation is sufficient. As per the equation, the ΔZ will become zero when the target velocity is equal to the present velocity i.e., when the target pressure distributions are equal to the present pressure distributions since pressure distributions are related to the velocity by the following relation in incompressible flow. Plugging the values for $d(\Delta Z)/dx$ and $d^2(\Delta Z)/dx^2$, the following equation is obtained:

$$A\Delta Z_i + B \frac{\Delta Z_{i+1} - \Delta Z_i}{x_{i+1} - x_i} - 2C \frac{\frac{\Delta Z_{i+1} - \Delta Z_i}{x_{i+1} - x_i} - \frac{\Delta Z_i - \Delta Z_{i-1}}{x_i - x_{i-1}}}{x_{i+1} - x_{i-1}} = V_{\text{Target},i}^2 - V_{\text{Actual},i}^2 \quad \text{Eq. (3)}$$

Using this equation, the given airfoil can be iterated (for both upper and lower surfaces) until the ΔZ values reach zero. The resulting airfoil shape would give us the target airfoil that meets the imposed pressure distribution requirements.

This methodology was applied using a Matlab code (provided in Appendix A) written by the author which customizes the MGM technique for each iteration, taking into account the new airfoil shape and corresponding coefficients for the differential equation, and produces an airfoil shape. By multiple iterations, the target airfoil is obtained. Figure 4 summarizes the implementation of the technique.

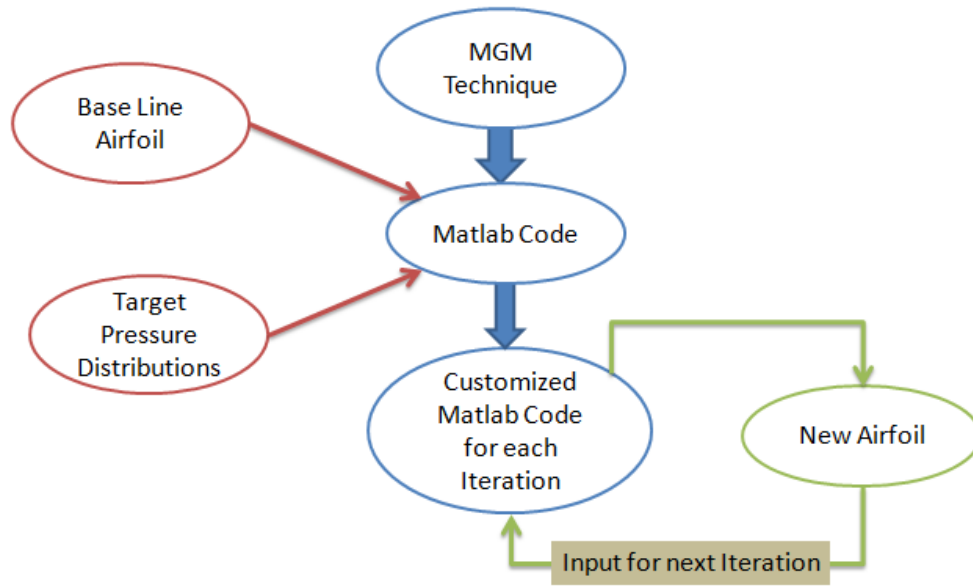


Figure 4: MGM technique implementation

Take-off Configuration Optimization

The developed optimization technique requires the translation of the common surface between the slat and the wing element to a polynomial (usually of fourth degree) as shown in Figure 5. This could be easily done by importing the points on the common surface into Matlab and using the built-in “polyfit” function to derive the equation that defines the line.

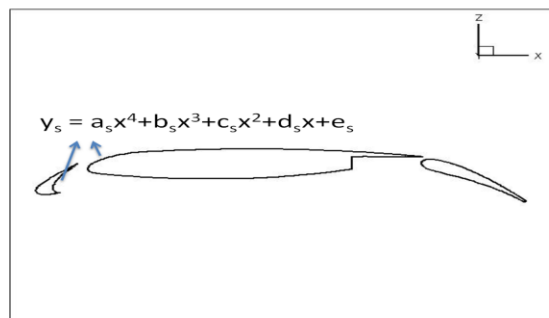


Figure 5: A sample of the numerical translation of the shape

After the equation defining the surface has been defined, the coefficients of the derived equation (a_s , b_s , c_s , d_s , e_s , etc.) were slightly varied to obtain a slightly different surface shape.

The points on the common surface were computed using the varied coefficients and a new surface was created. The original surface was replaced by the surface obtained by the new coefficients. Once a new shape was obtained, a grid was created around the airfoil which was fed into the flow solver to obtain the aerodynamic characteristics of the new airfoil shape. The coefficients of the surface equation were varied multiple times to study the effects of each of the coefficients on the overall airfoil aerodynamics. Through studying the individual effects of the coefficients of the surface equation, the coefficients were collectively varied such that the resulting configuration maximized lift produced. Drag is not of primary concern in this study, although it could be easily incorporated in future studies.

Results & Discussion

Steady-flight Configuration

The important step in using simulations is to pick reliable tools. As seen in previous researches, the reliability of the flow solvers is of primary concern in CFD work. For the single element airfoil case, XFOIL has been a popular tool used in many studies [15]. The tool is open-source and is developed by Mark Drela. Using the NACA 0012 airfoil, the results from the flow solver were compared with the empirical results [1]. Figures 6, 7, and 8 show a good agreement between the XFOIL results and the experimental data, proving the reliability of XFOIL for single-element airfoils.

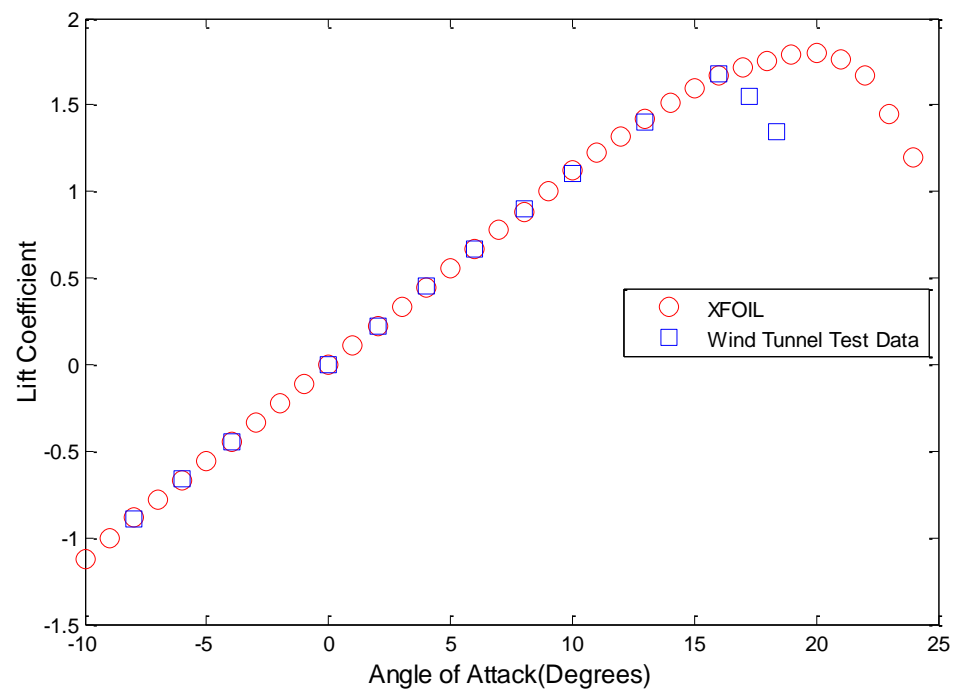


Figure 6: Comparison of C_L vs. AoA

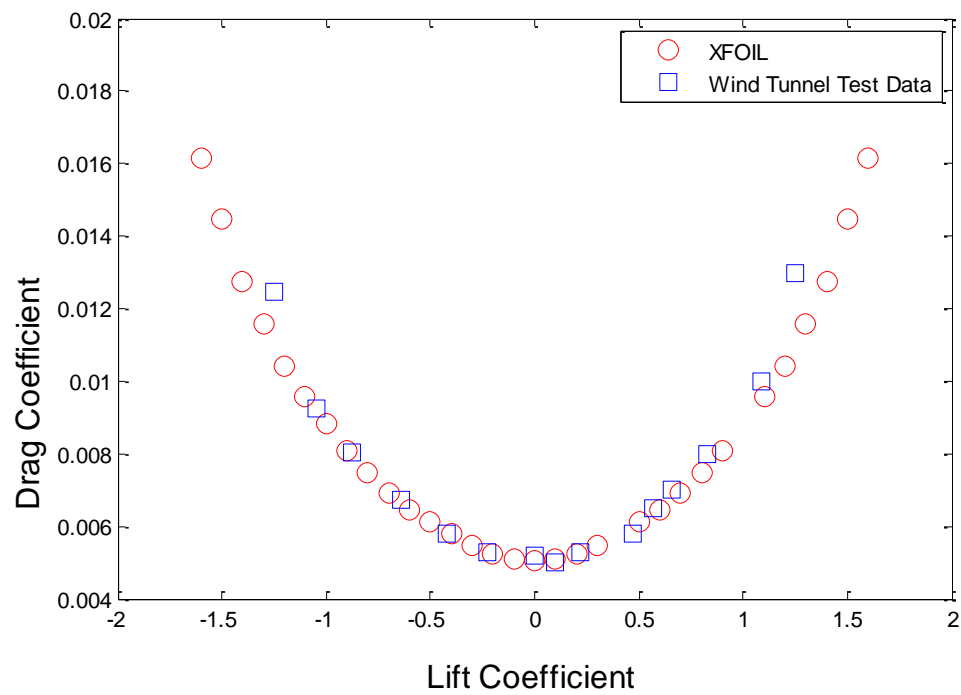


Figure 7: Comparison of C_D vs. C_L

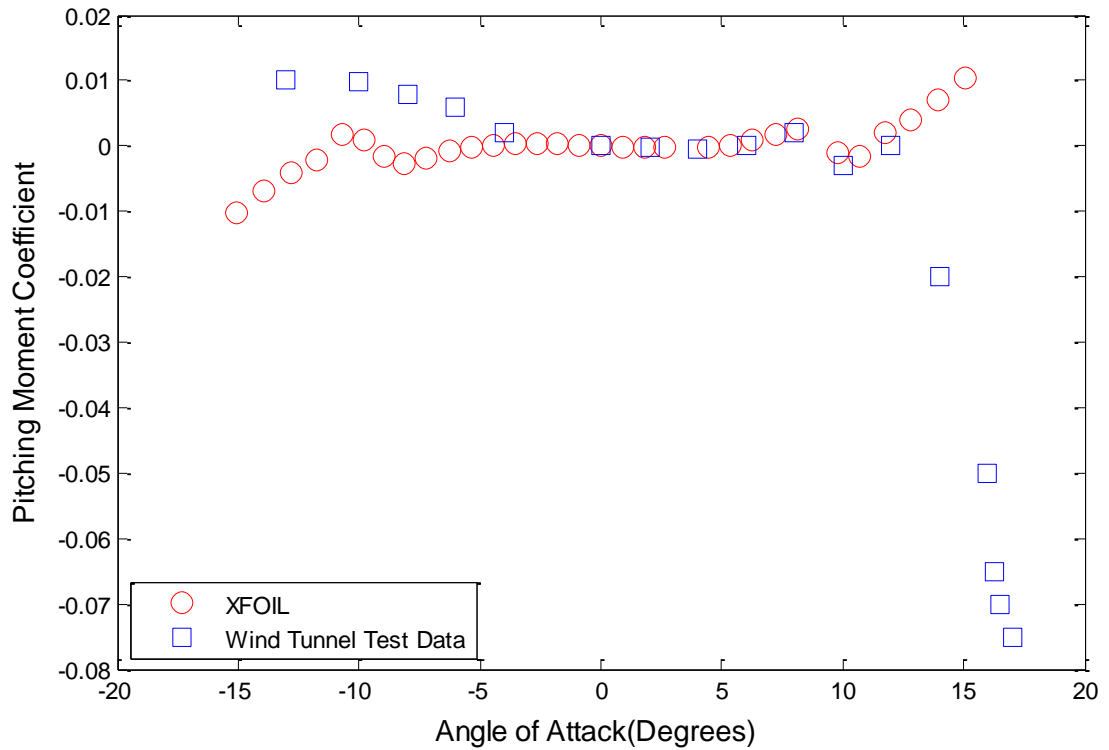


Figure 8: Comparison of C_m vs. α

Reduction in the drag coefficient value is the key to improving the performance of an airfoil. Choosing a laminar airfoil as the target airfoil in the MGM technique would result in reduction of drag coefficient of the baseline airfoil. For an airfoil to be considered a laminar flow airfoil, it must have a favorable pressure gradient that extends past 30% of the chord length. For NACA 0012 the laminar flow extends to about 5-20% of the airfoil length. An ideal laminar flow airfoil, Eppler 1200 in this case, has a laminar flow extended to 50-60% of the airfoil length. This extra presence of the laminar flow through the airfoil length results in a reduction of drag values. To obtain this result, the aforementioned process of extracting the airfoil properties from XFOIL and plugging them into the Matlab Code to execute the Modified Garabedian McFadden Technique is implemented. The process is repeated until the change in contour shapes over iterations became negligible. After 20 iterations, the following transformations in the airfoils were seen.

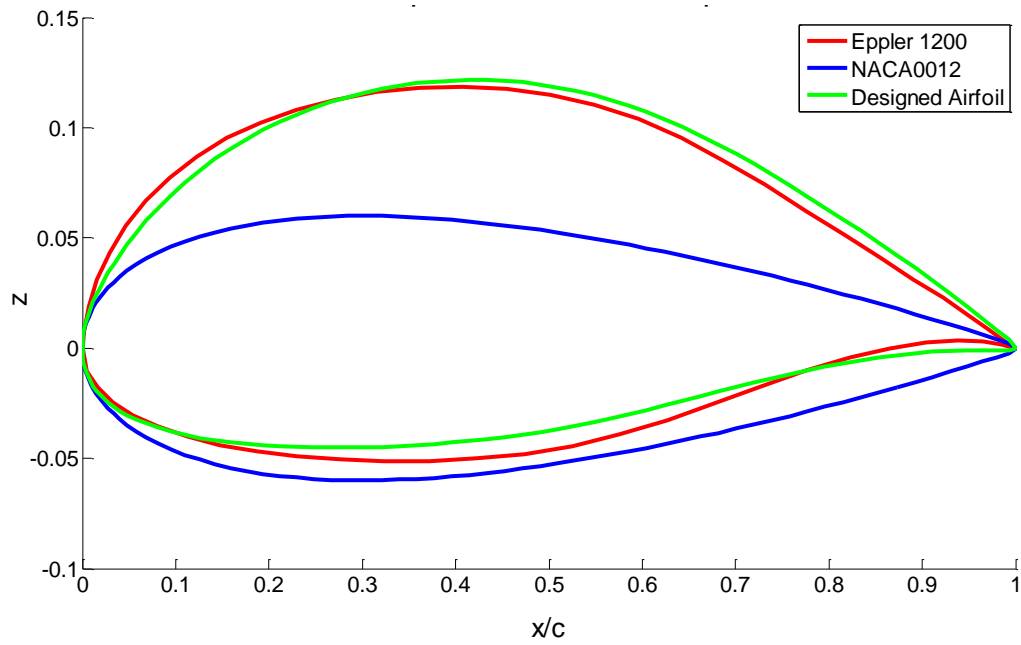


Figure 9: Transformation in the airfoil shape using MGM technique

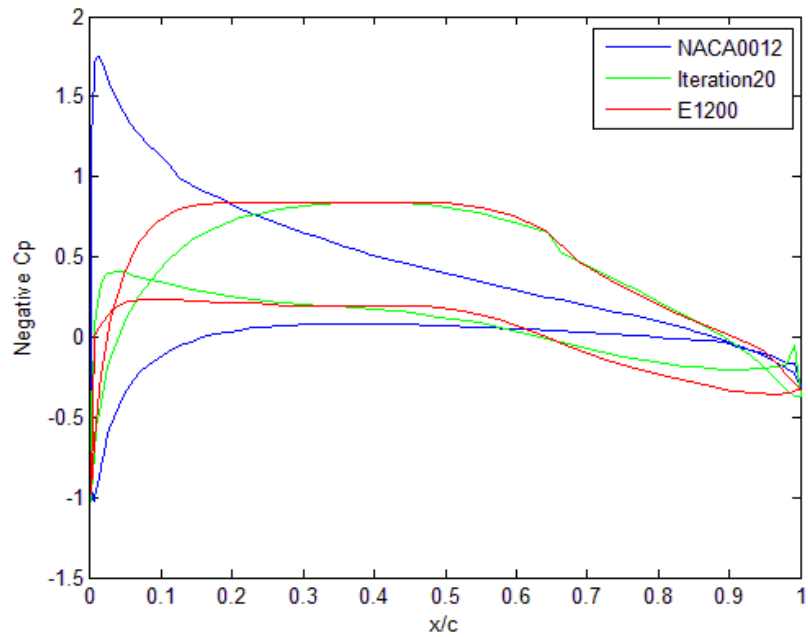


Figure 10 : Transformation seen in the Pressure distributions

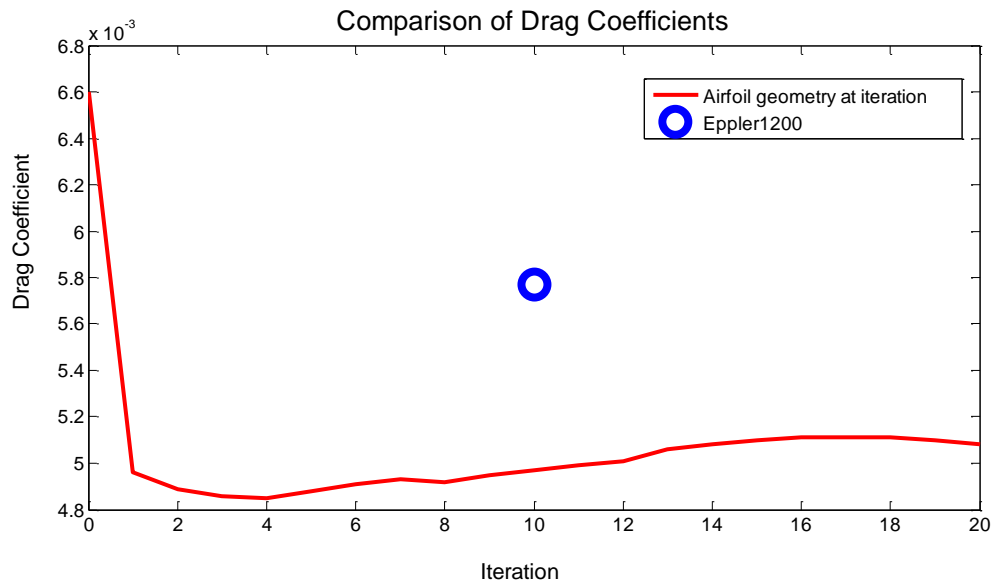


Figure 11: Cd values have changed significantly with the each iteration

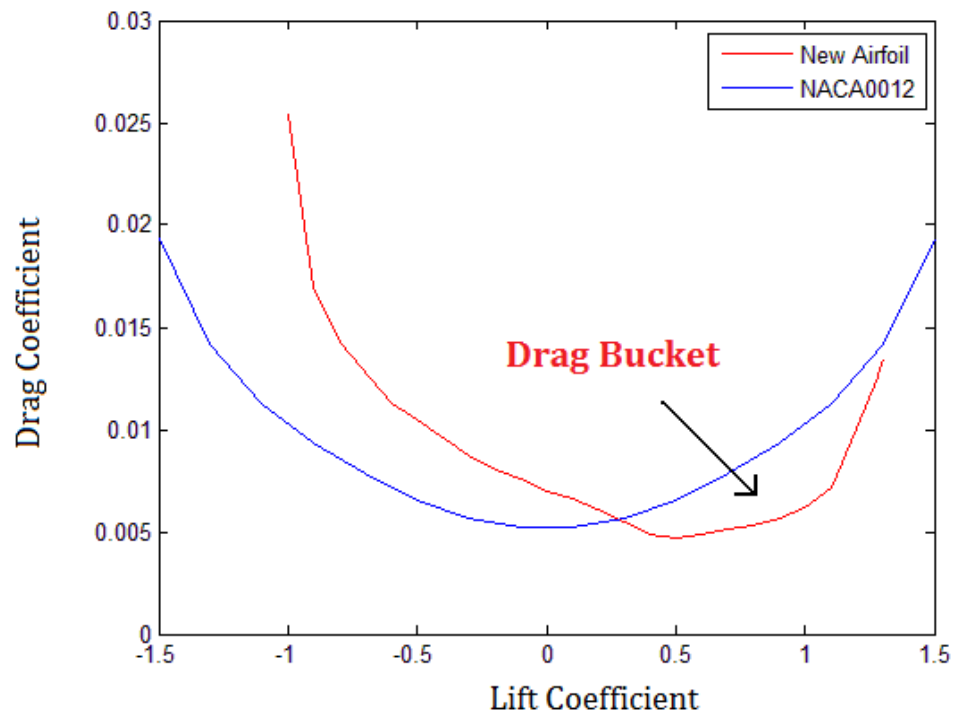


Figure 12: Designed airfoil's performance

The optimality of the new design is shown in the above presented plots. Figure 9 shows the comparison of the three airfoils shapes – baseline NACA 0012, target Eppler 1200, and the newly designed airfoil. The essence of the results lies in Figures 11 and 12. It is evident that newly designed airfoil offers lesser drag coefficient than not only the baseline airfoil but also the target Eppler 1200 airfoil. The performance of the airfoil as seen in Figure 12 shows the occurrence of drag bucket – the lowering of the drag for a range of lift coefficients which represents a range of flight conditions. The higher drag at negative lift represents that the airfoil produces higher opposing force while the flight is lowering altitude or landing, which is often preferred. While most design studies produce optimized results for narrow range of flight conditions, application of MGM technique has resulted in a design that provides optimal performance in almost all of the flight conditions. From these results, it can be concluded that the objective to design an optimal airfoil shape for steady flight condition has been met. The designed airfoil also has an identical profile to the design described in Venkataraman's paper discussed in literature review.

The MGM technique could also be applied to cases when the target pressure distributions could be developed based on the desired lift and drag values. Using these governing conditions, the airfoil that produces the desired lift and drag can be easily obtained with minimal use of computational resources and time.

Take-off Configuration

Due to the rarity of three-element airfoil data availability for academic purposes, an airfoil that was used in one of the AGARD conferences as a challenge problem – the L1T2 airfoil was used [2, 4]. Figure 13 shows the original configuration when the slat is deflected at 25° and the flap is deployed at 20° . The previously used XFOIL couldn't be used for this kind of configurations due to geometric complications. Therefore, commercial CFD tools such as

GridGen had to be used to construct the grids around the airfoils and FLUENT (flow solver) had to be used to obtain the aerodynamic properties. In order to validate the results from FLUENT, the obtained pressure coefficient (C_p) values were compared with the experimental values. Figure 12 shows the comparison of empirical and simulated pressure distributions of the original airfoil at 4.01° angle of attack, 0.197 Mach number, and a Reynolds number of 3.52×10^6 . K-epsilon turbulence model was used since it provided the best results compared to the wind tunnel data. The CFD results show a strong agreement with the wind tunnel test data for the C_p distributions around the airfoil and the flap. However, the vorticity near the lower surface of the airfoil was over predicted as seen in Figure 15. Although the over prediction of the vorticity had effects on the lift coefficients, the obtained values were only slightly lower than the test data and the error percentage was less than 10% for multiple angles of attack.

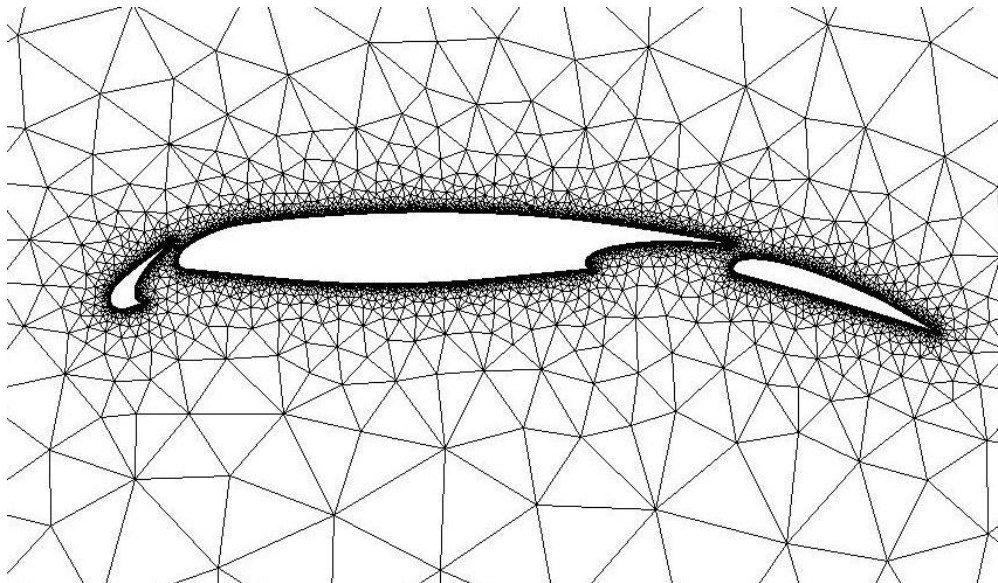


Figure 13: Original configuration of the AGARD airfoil

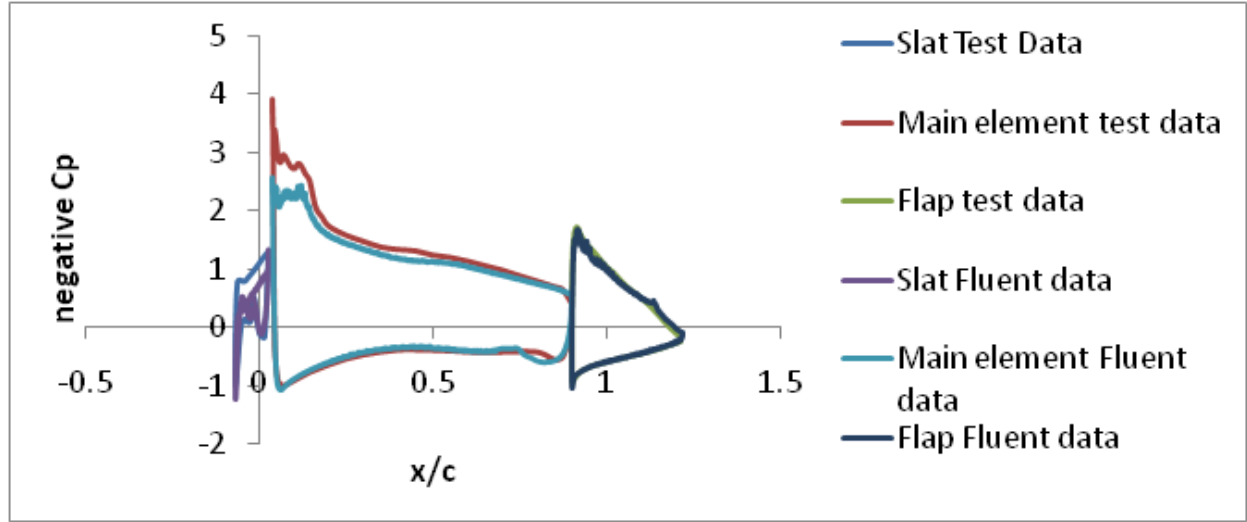


Figure 14: Comparison of pressure coefficients

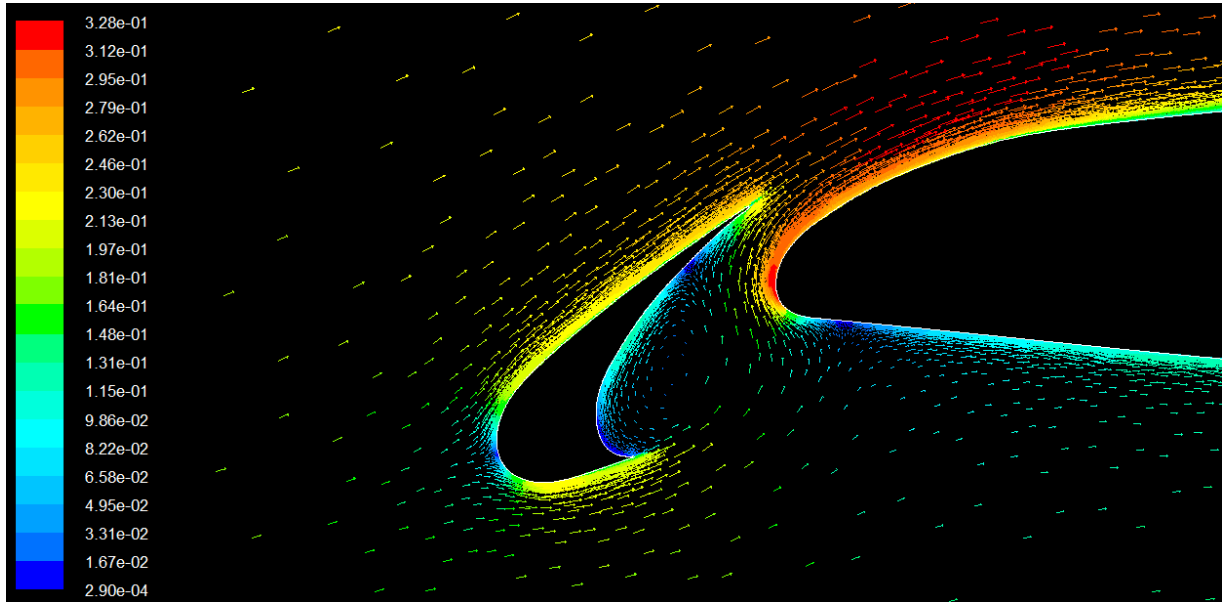


Figure 15: Over predicted flow reversal on the lower side of the Slat

As detailed in the methodology section, the surface between the slat and the wing element was translated to a fourth degree polynomial. The coefficients of this polynomial were varied slightly resulting in different surfaces shapes. The original common surface was defined by Eq. (4)

$$z = -1.0324 * 10^4 * x^4 + 0.4487 * 10^4 * x^3 - 0.0785 * 10^4 * x^2 + 0.0074 * 10^4 * x - 2 \quad \text{Eq. (4)}$$

The coefficients are relatively large because the x-values of the slat are small and z-values are comparatively very large. By perturbing these coefficients, different surfaces are obtained. In case 1, all coefficients are increased by 0.1, in case 2 by 0.15, in case 3 by 0.2, and in case 4 by -0.1. However, these don't make a large difference in the coefficients. Therefore, the individual sets of coefficients used for each case are not shown here. However, the differences between the four cases are plotted in Figure 16.

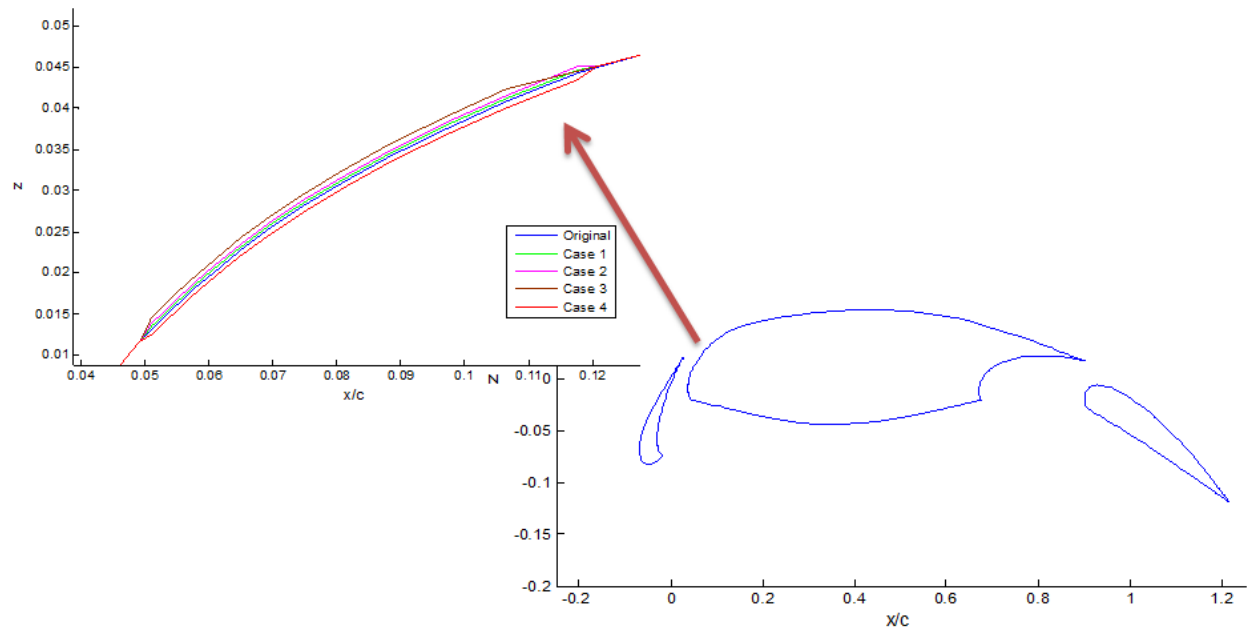


Figure 16: Visualization of the perturbed surfaces

Along with these minor perturbations, the slat gap i.e. the gap between the main element and the slat which was originally, 0.0625 was also decreased by 0.02 for each case. The new geometries were fed back into the CFD tools to obtain their aerodynamic properties and analyze the effects of the perturbations. The CFD results are presented in Table I.

Table I: Comparison of new and original cases

Case	Lift coefficient	Improvement (%)
Baseline	1.84	
Case 1	1.91	3.80
Case 2	1.96	6.52
Case 3	1.98	7.61
Case 4	1.8	-2.17

The results show the efficiency of the developed optimization technique. Cases 1-3 showed an increase in the lift values. Case 4 showed a decrease in lift showing that inward perturbation of the airfoil results in adverse pressures. It can be concluded that Case 3 is the most efficient configuration showing an increase of 7.61% in lift coefficient at the take-off conditions. While the results are not optimal as one would hope, they are significant because L1T2 is already a highly optimized airfoil. This design procedure required few computational resources and efficient solutions were obtained using very small perturbations. This method could be developed further to ensure optimal solutions at each trial which would cut short the time required to produce the design.

Conclusions and Recommendations

The application of the MGM technique to the single-element configuration resulted in a drag bracket for the design airfoil resulting from NACA 0012. The L1T2 airfoil was used for the application of the optimization technique developed for the three-element configuration. The results show an optimum airfoil which resulted in increased lift. Future work could be conducted on extension of the MGM technique to 3D wings and fuselages. Similarly, the optimization technique could be developed to apply for airfoils with higher number of elements. It could also be applied to the flap configuration and efficient results could be obtained. With the ongoing

research in turbulence models, more reliable CFD tools are being produced. Use of this sophisticated technology can increase the reliability and practicality of the results.

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Appendix A

Matlab Code to execute MGM technique

```
%%defining constants
A = 6;
B = 6;
C = 6;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%LOWER SURFACE
%%extracting all the X values
xlower = xlsread('i.xls','A51:A100')
%%extracting all the Z values
zlower = xlsread('i.xls','B51:B100')
%%extracting RHS matrix
RHSlower = xlsread('i.xls','G51:G100')
RHSlower(1,1) = 0;
RHSlower(50,1) = 0;
%%defining the LHS matrix
LHSlower = zeros(50,50)
LHSlower(1,1) = 1;
LHSlower(50,50) = 1;
for i = 2:49
    %defining the main diagonal
    LHSlower(i,i) = A - (B/(xlower(i+1)-xlower(i))) + (C/((xlower(i+1)-
xlower(i-1))/2))*((1/(xlower(i+1)-xlower(i)))+(1/(xlower(i)-xlower(i-1)))));
    %defining the lower diagonal
    LHSlower(i,i-1) = -(C/((xlower(i+1)-xlower(i-1))/2))*(1/(xlower(i)-
xlower(i-1)));
    %defining the upper diagonal
    LHSlower(i,i+1) = (B/(xlower(i+1)-xlower(i))) - (C/((xlower(i+1)-
xlower(i-1))/2))*(1/(xlower(i+1)-xlower(i)));
end;
%%computing deltaZ values
deltaZlower = LHSlower\RHSlower
%%adding deltaZ values to the Z matrix to get the co-ordinates for the new
%%airfoil
newZlower = zeros(50,1);
newZlower = zlower - deltaZlower
newZlower(1,1) = 0;
newZlower(50,1) = 0;
%writing these values to an excel sheet
output = xlswrite('iteration.xls',xlower,'A51:A100')
output = xlswrite('iteration.xls',zlower,'B51:B100')
output = xlswrite('iteration.xls',newZlower,'C51:C100')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%UPPER SURFACE
%%extracting all the X values
xupper = xlsread('i.xls','A2:A51')
%%extracting all the Z values
zupper = xlsread('i.xls','B2:B51')
%%extracting RHS matrix
RHSupper = xlsread('i.xls','G2:G51')
RHSupper(1,1) = 0;
```

```

RHSupper(50,1) = 0;
%%defining the LHS matrix
LHSupper = zeros(50,50)
LHSupper(1,1) = 1;
LHSupper(50,50) = 1;
for i = 2:49
    %defining the main diagonal
    LHSupper(i,i) = A - (B/(xupper(i+1)-xupper(i))) + (C/((xupper(i+1)-
xupper(i-1))/2))*((1/(xupper(i+1)-xupper(i)))+(1/(xupper(i)-xupper(i-1)))));
    %defining the lower diagonal
    LHSupper(i,i-1) = -(C/((xupper(i+1)-xupper(i-1))/2))*((1/(xupper(i)-
xupper(i-1)))));
    %defining the upper diagonal
    LHSupper(i,i+1) = (B/(xupper(i+1)-xupper(i))) - (C/((xupper(i+1)-
xupper(i-1))/2))*((1/(xupper(i+1)-xupper(i)))));
end;
%%computing deltaZ values
deltaZupper = LHSupper\RHSupper
%%adding deltaZ values to the Z matrix to get the co-ordinates for the new
%%airfoil
newZupper = zeros(50,1);
newZupper = zupper + deltaZupper
newZupper(1,1) = 0;
newZupper(50,1) = 0;
% %writing these values to an excel sheet
output = xlswrite('iteration.xls',xupper,'A2:A51')
output = xlswrite('iteration.xls',zupper,'B2:B51')
output = xlswrite('iteration.xls',newZupper,'C2:C51')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%extracting data from target airfoil
naca0012 = dlmread('naca0012.dat')
naca0012x = naca0012(:,1)
naca0012z = naca0012(:,2)
naca4412 = dlmread('e1200.dat')
naca4412x = naca4412(:,1)
naca4412z = naca4412(:,2)
ilx = xlsread('iteration.xls','A2:A100')
ilz = xlsread('iteration.xls','C2:C100')
%plotting the new and old airfoil along with the target airfoil
plot(naca0012x,naca0012z,'b')
hold on
plot(naca4412x,naca4412z,'r')
hold on
plot(ilx,ilz,'c')
legend('NACA0012','Eppler1200','Iteration1')

```